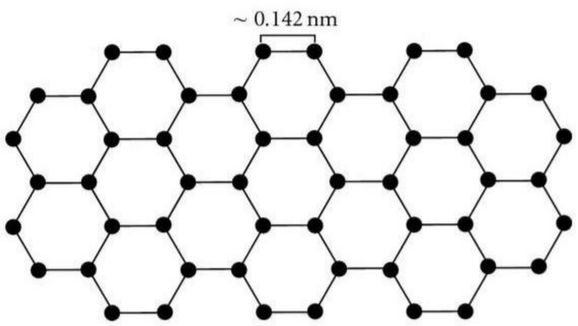
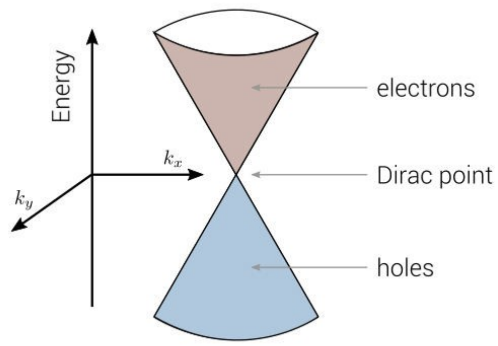
HOMEWORK SET 05
Theory of Condensed Matter
UFV/TKL1/99 lecture by Martin Gmitra
Winter Semester 2024, room KNKTFA

1. Consider an energy band with the following dispersion
$E(\mathbf{k}) = \mathrm{const.} + \hbar^2 \left(\frac{k_x^2}{2m_x} + \frac{k_y^2}{2m_y} + \frac{k_z^2}{2m_z} \right)$ and
   a) [*2 points*] calculate density of states and electronic specific heat. By comparing result of the electronic specific heat for free electrons, show that the specific heat effective mass is given by $m^* = (m_x m_y m_z)^{1/3}$.
   b) [*2 points*] Solve the semi-classical equation of motion $m_\alpha \frac{\mathrm{d}v_\alpha}{\mathrm{d}t} = -e[E_\alpha + (\mathbf{v} \times \mathbf{B})_\alpha]$, where $\alpha = x, y, z$. Assuming for the electric field $\mathbf{E} = 0$, and the magnetic field $\mathbf{B} = B\mathbf{e}_z$ show that the cyclotron frequency is given by $\omega_c = eB/m_c^*$, where the cyclotron effective mass is $m_c^* = (m_x m_y)^{1/2}$.

2. [*2 points*] Verify that $\phi_{n,k_y}(x) = \frac{1}{\sqrt{2^n n! 2\pi \ell^2}} H_n[(x - k_y \ell^2)/\ell] e^{-(x-k_y\ell^2)^2/2\ell^2}$, where $H_n(x)$ are the Hermite polynomials, is a solution of the Schrödinger equation with Hamiltonian
$H = \frac{p_x^2}{2m} + \frac{1}{2m}(\hbar k_y - eBx)^2$.

3. Calculate the specific heat of graphene at half filling. The perfect particle-hole symmetry fixes chemical potential to the Dirac point at all temperatures, see figure below for two-dimensional structure and energy band dispersion of graphene.





   a) [*2 points*] For the band structure of graphene consider the linear dispersion $\epsilon_{\mathbf{k}} = \pm\hbar v_F|\mathbf{k}|$, where the Fermi velocity $v_F = 3ta/2\hbar \simeq 10^6\,\mathrm{m/s}$, with hopping $t = 2.8\,\mathrm{eV}$ and $a = 1.42\,\text{Å}$ is the carbon-carbon distance. The density of states $\rho(E) = \frac{3\sqrt{3}a^2|E|}{\hbar^2 \pi v_F^2}$.
   b) [*2 points*] Calculate shift of the chemical potential if graphene is doped by the electrons of concentration of $0.73\ \times 10^{12}\mathrm{cm}^{-2}$.

4. [*4 extra points*] The Pauli spin susceptibility is defined $\chi_\mathrm{P} = \left(\frac{\partial M}{\partial H}\right)$ where M is the magnetization. Calculate the Pauli spin susceptibility for graphene.

5. [*6 extra points*] Consider two dimensional square lattice. By means of tight-binding model calculate (numerically writing a computer program) effect of the magnetic field perpendicular to the lattice plane. Use Landau gauge and Peierls phase factor for nearest-neighbor hopping only.

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