HOMEWORK SET 08

Theory of Condensed Matter

UFV/TKL1/99 lecture by Martin Gmitra

Winter Semester 2021, room KNKTFA(Pa9-PKn)

1. Derive complex refractive index for homogeneous non-magnetic material ($\mathbf{B} = \mathbf{H}$) in absence of external charges and currents. Consider Maxwell's equation for electromagnetic transverse waves propagating along the z-direction in a medium and interacting with the internal charge density and internal current density $\mathbf{J}(\mathbf{r}, t)$ with electric field along the x-direction $\mathbf{E}(\mathbf{r}, t) = E(z)e^{-i\omega t}\hat{\mathbf{e}}_x$, magnetic field along the y-direction $\mathbf{B}(\mathbf{r}, t) = B(z)e^{-i\omega t}\hat{\mathbf{e}}_y$, and current density along the x-direction $\mathbf{J}(\mathbf{r}, t) = J(z)e^{-i\omega t}\hat{\mathbf{e}}_x$. For current density assume local response regime with frequency dependent conductivity $\mathbf{J}(\mathbf{r}) = \sigma(\omega)\mathbf{E}(\mathbf{r})$.

   a) [*2 points*] Show that the complex refractive index $\tilde{n}^2 = 1 + \dfrac{i\sigma(\omega)}{\omega\epsilon_0}$ in SI units

   b) [*1 point*] and $\tilde{n}^2 = 1 + \dfrac{4\pi i\sigma(\omega)}{\omega}$ in Gauss units.

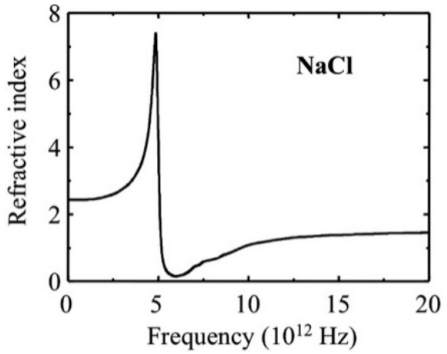
2. [*1 point*] Consider an incoherent light passing through a solid of thickness ℓ with two surfaces with reflectivity R_1 and R_2. Show that the transmissivity is equal

$$T = \frac{(1 - R_1)(1 - R_2)e^{-\alpha\ell}}{1 - R_1 R_2 e^{-2\alpha\ell}}.$$

3. [*1 point*] The conductivity of aluminium at room temperature is $4.1 \times 10^7 \Omega^{-1}\mathrm{m}^{-1}$. Free electron like density of for aliuminiun is $1.81 \times 10^{29}\mathrm{m}^{-3}$ and plasma frequency $\Omega_\mathrm{p} = 2.4 \times 10^{16}\mathrm{s}^{-1}$. Calculate the reflectivity at $500\,\mathrm{nm}$ according to the Drude–Lorentz model.

4. [*1 point*] Plot the refractive index, the real part n and imaginary part $\varkappa$, as a function of frequency using Lorentz oscillator model with the characteristic frequency $\omega_0 = 10^{14}\,\mathrm{s}^{-1}$, damping $\gamma = 5 \times 10^{12}\,\mathrm{s}^{-1}$, the static dielectric constant $\varepsilon_\mathrm{static} = 12$ and the infinite frequency constant $\varepsilon_\infty = 10$.

5. [*2 points*] Below is shown refractive index of NaCl, estimate Lorentz's oscillator model parameters, static dielectric constant, characteristic frequency ω_0, and damping γ.



6. [*1 point*] Calculate skin depth δ at 50 Hz and 100 MHz for cooper which dc electrical conductivity is $6.5 \times 10^7\,\Omega^{-1}\mathrm{m}^{-1}$.

7. [*1 point*] The sea water had dc conductivity of about $4\,\Omega^{-1}\mathrm{m}^{-1}$. Suppose you plan to transmit radio waves of frequency 200 kHz to a submarine. How far will the waves go?

HOMEWORK SET 08

Theory of Condensed Matter

UFV/TKL1/99 lecture by Martin Gmitra

Winter Semester 2021, room KNKTFA(Pa9-PKn)