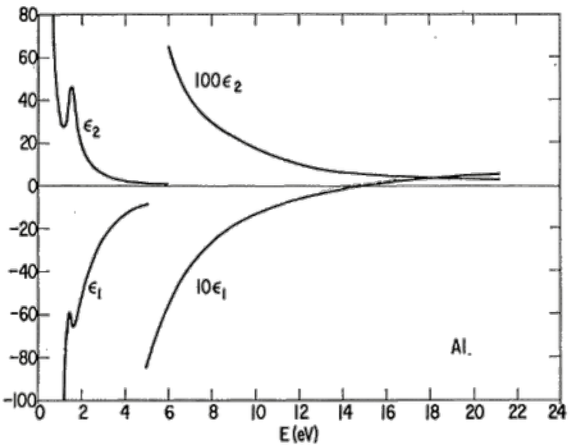
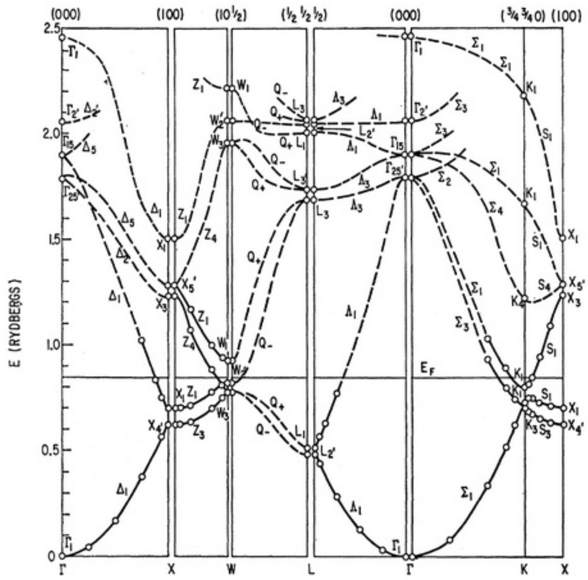
HOMEWORK SET 09
Theory of Condensed Matter
UFV/TKL1/99 lecture by Martin Gmitra
Winter Semester 2024, room: KNKTFA

1. [*3 points*] Consider three dimensional system of fully occupied valence free electron like band and a free electron like band separated by the energy gap of $0.18\,\mathrm{eV}$. Assume for the valence electrons effective mass $m_\mathrm{v}^* = 0.4\,m_0$ and for the conduction electrons the mass $m_\mathrm{e}^* = 0.014\,m_0$, where m_0 is the free electron mass. How much energy from a $0.5\,\mathrm{eV}$photon is converted into free electron kinetic energy, and how much is converted into the free hole energy?

2. [*2 points*] Prove that if a uniform electron gas is displaced by a small amount in one direction relative to the uniform background of compensating positive charge and then is allowed to relax, it will oscillate with the plasma frequency, $\Omega_\mathrm{p} = \dfrac{e^2 n}{\varepsilon_0 m}$, where $n = N/V$ is the electron gas density. The oscillation modes describing the response on the displacement are called *plasmons*.

3. Figures below show dielectric function and band structure of fcc aluminium.
   a) [*2 points*] Extract plasmon frequency and calculate the average radius r_s of the sphere corresponding to the average volume per electron and compare to the Wigner-Seitz radius knowing the lattice constant $4.046\,\mathrm{\AA}$.
   b) [*1 point*] The bound-electron contribution at about 1.4 eV is related to the specific optical transition around which high symmetry points?





4. [*2 points*] Using the following mathematical identity $\dfrac{1}{x - i\eta} = \dfrac{x}{x^2+\eta^2} + i\left(\dfrac{\eta}{x^2+\eta^2}\right)$and $\lim\limits_{\eta\to 0^+} \dfrac{x}{x^2+\eta^2} = \mathcal{P}\dfrac{1}{x}$, $\lim\limits_{\eta\to 0^+}\left(\dfrac{\eta}{x^2+\eta^2}\right) = \pi\delta(x)$, find real and imaginary components of the dielectric function $\varepsilon(\mathbf{q},\omega) = 1 - \dfrac{e^2}{Vq^2}\sum\limits_{\kappa,\kappa'} \dfrac{|\langle\psi_{\kappa'}|e^{i\mathbf{q}\cdot\mathbf{r}}|\psi_\kappa\rangle|^2}{\epsilon_{\kappa'} - \epsilon_\kappa - \hbar(\omega + i\eta)}(f_{\kappa'} - f_\kappa)$.

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