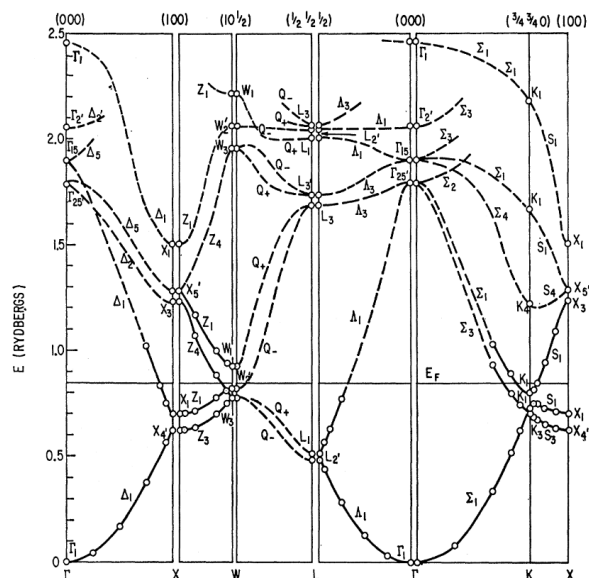
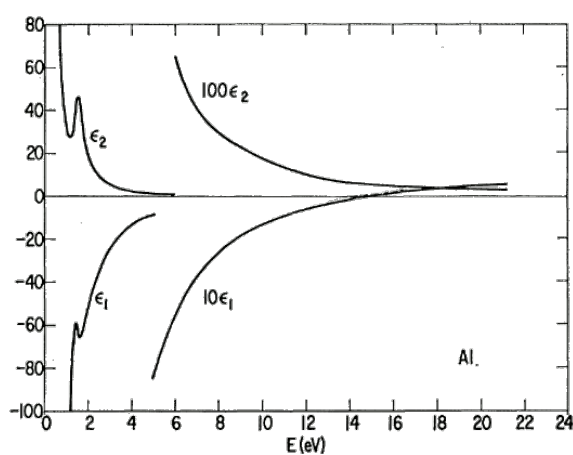


HOMEWORK SET 09
 Theory of Condensed Matter
 UFV/TKL1/99 lecture by Martin Gmitra
 Winter Semester 2021, room: distant due COVID-19

1. [3 points] Consider three dimensional system of fully occupied valence free electron like band and a free electron like band separated by the energy gap of 0.18 eV. Assume for the valence electrons effective mass $m_v^* = 0.4 m_0$ and for the conduction electrons the mass $m_c^* = 0.014 m_0$, where m_0 is the free electron mass. How much energy from a 0.5 eV photon is converted into free electron kinetic energy, and how is converted into the free hole energy?
2. [2 points] Prove that if a uniform electron gas is displaced by a small amount in one direction relative to the uniform background of compensating positive charge and then is allowed to relax, it will oscillate with the plasma frequency, $\Omega_p = \frac{e^2 n}{\epsilon_0 m}$, where $n = N/V$ is the electron gas density. The oscillation modes describing the response on the displacement are called *plasmons*.
3. Figures below show dielectric function and band structure of fcc aluminium.
 - a) [2 points] Extract plasmon frequency and calculate the average radius r_s of the sphere corresponding to the average volume per electron and compare to the Wigner-Seitz radius knowing the lattice constant 4.046 Å.
 - b) [1 point] The bound-electron contribution at about 1.4 eV is related to the specific optical transition around which high symmetry points?



4. [2 points] Using the following mathematical identity $\frac{1}{x - i\eta} = \frac{x}{x^2 + \eta^2} + i \left(\frac{\eta}{x^2 + \eta^2} \right)$ and $\lim_{\eta \rightarrow 0^+} \frac{x}{x^2 + \eta^2} = \mathcal{P} \frac{1}{x}$, $\lim_{\eta \rightarrow 0^+} \left(\frac{\eta}{x^2 + \eta^2} \right) = \pi \delta(x)$, find real and imaginary components of the dielectric function $\epsilon(\mathbf{q}, \omega) = 1 - \frac{e^2}{V q^2} \sum_{\kappa, \kappa'} \frac{|\langle \psi_{\kappa'} | e^{i\mathbf{q} \cdot \mathbf{r}} | \psi_{\kappa} \rangle|^2}{\epsilon_{\kappa'} - \epsilon_{\kappa} - \hbar(\omega + i\eta)} (f_{\kappa'} - f_{\kappa})$.

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