HOMEWORK SET 10

Theory of Condensed Matter UFV/TKL1/99 lecture by Martin Gmitra Winter Semester 2024, room: KNKTFA

- 1. Show that current density operator $\mathbf{j} = \frac{1}{2m} \left[((\mathbf{p} q\mathbf{A})\psi)^*\psi + \psi^*(\mathbf{p} q\mathbf{A})\psi \right]$ for the so called Ginzburg-Landau wavefunction or order parameter of the form $\psi(\mathbf{r}) = \sqrt{\rho(\mathbf{r})}e^{i\varphi(\mathbf{r})}$
 - a) [1 point] equals $\mathbf{j} = \frac{\hbar}{m} \left[\nabla \varphi(\mathbf{r}) \frac{q}{\hbar} \mathbf{A} \right] \rho(\mathbf{r}).$
 - b) [3 points] assume that in the bulk material the current density vanishes, so that $\hbar \nabla \varphi(\mathbf{r}) = q \mathbf{A}$. Integrate both the sides of the expression around a closed loop in a superconducting ring and show that the resulting magnetic flux is quantized. What is the correct value of charge q for a superconductor?
- 2. Temperature dependence of the critical field of a type-I superconductor is given by

$$B_{\rm c} = B_0 \left[1 - \left(\frac{T}{T_{\rm c}} \right)^2 \right]$$
. Use this to show that

- a) [2 points] entropy S_s per volume of the superconducting state is lower than that of the normal state $S_{\rm n}$.
- b) [2 points] show that $S_n S_s = aT bT^3$, where a and b are constants, and that $S_n = S_s$ when $T = T_c$.
- 3. [2 points] A vortex in a superconductor can be modeled as having a cylidrical core of normal metal of radius ξ_0 . Use London's equation in cylindrical coordinates to show that the magnetic field $B_z(r)$ outside of the core obeys $\frac{1}{r}\frac{d}{dr}\left(r\frac{dB_z}{dr}\right) = \frac{B_z}{\lambda^2}$, where λ is the

penetration depth.

- 4. [2 extra points] Find the Fourier transform of the Coulomb potential $V(r) = e^2/r$. Hint: consider the Yukawa potential $V_{\lambda}(r) = \frac{e^2 \exp(-\lambda r)}{r}$ and set at the end $\lambda \to 0$.
- 5. [2 extra points] Derive general form of the electron-phonon matrix element for small displacement. Hint: follow derivation in the shared textbook.
- 6. [2 extra points] Attractive electron-electron interaction caused by a virtual exchange of the phonons favors formation of a bound electron pair when $|\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}+\mathbf{q}}| < \hbar\omega(\mathbf{q})$. In the case of $|\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}+\mathbf{q}}| = \hbar\omega(\mathbf{q})$ the expression for the energy would have a pole (diverges). Consider realistic phonon energy of 1THz that correspond to 4.13567 meV. Assume acoustic phonon modes with $\omega(\mathbf{q}) = vq$ where speed of sound is 6300 m/s corresponding to aluminium. What would be the lattice constant of the aluminium when considering free-electron like dispersion $\hbar^2 k^2$ $\epsilon_{\mathbf{k}}$

$$c = \frac{1}{2m}$$

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