HOMEWORK SET 10

Theory of Condensed Matter
UFV/TKL1/99 lecture by Martin Gmitra
Winter Semester 2021, room: distant due COVID-19

1. Show that current density operator $\mathbf{j} = \frac{1}{2m}\left[((\mathbf{p}-q\mathbf{A})\psi)^*\psi + \psi^*(\mathbf{p}-q\mathbf{A})\psi\right]$ for the so called Ginzburg-Landau wavefunction or order parameter of the form $\psi(\mathbf{r}) = \sqrt{\rho(\mathbf{r})}e^{i\varphi(\mathbf{r})}$
   a) [*1 point*] equals $\mathbf{j} = \frac{\hbar}{m}\left[\nabla\varphi(\mathbf{r}) - \frac{q}{\hbar}\mathbf{A}\right]\rho(\mathbf{r})$.
   b) [*3 points*] assume that in the bulk material the current density vanishes, so that $\hbar\nabla\varphi(\mathbf{r}) = q\mathbf{A}$. Integrate both the sides of the expression around a closed loop in a superconducting ring and show that the resulting magnetic flux is quantized. What is the correct value of charge q for a superconductor?

2. Temperature dependence of the critical field of a type-I superconductor is given by $B_\mathrm{c} = B_0\left[1 - \left(\frac{T}{T_\mathrm{c}}\right)^2\right]$. Use this to show that
   a) [*2 points*] entropy S_s per volume of the superconducting state is lower than that of the normal state S_n.
   b) [*2 points*] show that $S_\mathrm{n} - S_\mathrm{s} = aT - bT^3$, where a and b are constants, and that $S_\mathrm{n} = S_\mathrm{s}$ when $T = T_\mathrm{c}$.

3. [*2 points*] A vortex in a superconductor can be modeled as having a cylidrical core of normal metal of radius ξ_0. Use London's equation in cylindrical coordinates to show that the magnetic field $B_z(r)$ outside of the core obeys $\frac{1}{r}\frac{d}{dr}\left(r\frac{dB_z}{dr}\right) = \frac{B_z}{\lambda^2}$, where λ is the penetration depth.

4. [*2 extra points*] Find the Fourier transform of the Coulomb potential $V(r) = e^2/r$. *Hint: consider the Yukawa potential* $V_\lambda(r) = \frac{e^2\exp(-\lambda r)}{r}$ *and set at the end* $\lambda \to 0$.

5. [*2 extra points*] Derive general form of the electron-phonon matrix element for small displacement. *Hint: follow derivation in the shared textbook.*

6. [*2 extra points*] Attractive electron-electron interaction caused by a virtual exchange of the phonons favors formation of a bound electron pair when $|\epsilon_\mathbf{k} - \epsilon_{\mathbf{k}+\mathbf{q}}| < \hbar\omega(\mathbf{q})$. In the case of $|\epsilon_\mathbf{k} - \epsilon_{\mathbf{k}+\mathbf{q}}| = \hbar\omega(\mathbf{q})$ the expression for the energy would have a pole (diverges). Consider realistic phonon energy of 1THz that correspond to 4.13567 meV. Assume acoustic phonon modes with $\omega(\mathbf{q}) = vq$ where speed of sound is 6300 m/s corresponding to aluminium. What would be the lattice constant of the aluminium when considering free-electron like dispersion $\epsilon_\mathbf{k} = \frac{\hbar^2k^2}{2m}$?

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