

# Appendix A

## SI Units and Gaussian Units

### A.1 Conversion of Amounts

All factors of 3 (apart from exponents) should, for accurate work, be replaced by 2.99792456, arising from the numerical value of the velocity of light. [43]

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Physical quantity	Symbol	SI(MKSA)		Gaussian
Length	$l$	1 meter (m)	$10^2$	centimeters (cm)
Mass	$m$	1 kilogram (kg)	$10^3$	grams (gm or g)
Time	$t$	1 second (sec or s)	1	second (sec or s)
Frequency	$f$	1 hertz (Hz)	1	hertz (Hz)
Force	$F$	1 newton (N)	$10^5$	dynes
Work, Energy	$W, U$	1 joule (J)	$10^7$	ergs
Power	$P$	1 watt (W)	$10^7$	ergs/s
Charge	$q$	1 coulomb (C)	$3 \times 10^9$	statcoulombs
Charge density	$\rho$	1 C/m <sup>3</sup>	$3 \times 10^3$	statcoul/cm <sup>3</sup>
Current	$I$	1 ampere (A)	$3 \times 10^9$	statamperes
Current density	$J$	1 A/m <sup>2</sup>	$3 \times 10^5$	statamp/cm <sup>2</sup>
Potential	$\varphi$	1 volt (V)	$10^{-2}/3$	statvolt
Electric field	$E$	1 V/m	$10^{-4}/3$	statvolt/cm
Electric induction	$D$	1 C/m <sup>2</sup>	$12\pi \times 10^5$	statvolt/cm
Polarization	$P$	1 C/m <sup>2</sup>	$3 \times 10^5$	moment/cm <sup>3</sup>
Magnetic flux	$\Phi$	1 weber (Wb)	$10^8$	maxwell (Mx)
Magnetic induction	$B$	1 tesla (T)	$10^4$	gauss (Gs)
Magnetic field	$H$	1 A/m	$4\pi \times 10^{-3}$	oersted (Oe)
Magnetization	$M$	1 A/m	$10^{-3}$	moment/cm <sup>3</sup>
Conductance	$G$	1 siemens (S)	$9 \times 10^{11}$	cm/s
Conductivity	$\sigma$	1 S/m	$9 \times 10^9$	1/s
Resistance	$R$	1 ohm ( $\Omega$ )	$10^{-11}/9$	s/cm
Capacitance	$C$	1 farad (F)	$9 \times 10^{11}$	cm
Inductance	$L$	1 henry (H)	$10^9$	cm

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## A.2 Formulas in SI (MKSA) Units and Gaussian Units

Name of formula	SI (MKSA)	Gaussian
Maxwell equations	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}$
	$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$	$\nabla \times \mathbf{H} = \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c} \mathbf{J}$
	$\nabla \cdot \mathbf{D} = \rho$	$\nabla \cdot \mathbf{D} = 4\pi \rho$
	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
Lorentz force	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$	$\mathbf{F} = q(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B})$
Constitutional equations	$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon \mathbf{E}$ $\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) = \mu \mathbf{H}$ $\mathbf{J} = \gamma \mathbf{E}$	$\mathbf{D} = \mathbf{E} + 4\pi \mathbf{P} = \epsilon \mathbf{E}$ $\mathbf{B} = \mathbf{H} + 4\pi \mathbf{M} = \mu \mathbf{H}$ $\mathbf{J} = \gamma \mathbf{E}$
Constitutional parameters	$\epsilon = \epsilon_0 (1 + \chi_e) = \epsilon_0 \epsilon_r$ $\mu = \mu_0 (1 + \chi_m) = \mu_0 \mu_r$	$\epsilon = 1 + 4\pi \chi_e = \epsilon_r$ $\mu = 1 + 4\pi \chi_m = \mu_r$
Boundary equations	$\mathbf{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0$ $\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{J}_s$ $\mathbf{n} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = \rho_s$ $\mathbf{n} \cdot (\mathbf{B}_2 - \mathbf{B}_1) = 0$	$\mathbf{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0$ $\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \frac{4\pi}{c} \mathbf{J}_s$ $\mathbf{n} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = 4\pi \rho_s$ $\mathbf{n} \cdot (\mathbf{B}_2 - \mathbf{B}_1) = 0$
Coulomb's law	$\mathbf{E} = \frac{1}{4\pi\epsilon} \int_V \frac{\rho}{r^2} \hat{\mathbf{r}} dV'$	$\mathbf{E} = \frac{1}{\epsilon} \int_V \frac{\rho}{r^2} \hat{\mathbf{r}} dV'$
	$\varphi = \frac{1}{4\pi\epsilon} \int_V \frac{\rho}{r} dV'$	$\varphi = \frac{1}{\epsilon} \int_V \frac{\rho}{r} dV'$
Biot-Savart law	$\mathbf{B} = \frac{\mu}{4\pi} \int_V \frac{\mathbf{J} \times \hat{\mathbf{r}}}{r^2} dV'$	$\mathbf{B} = \frac{\mu}{c} \int_V \frac{\mathbf{J} \times \hat{\mathbf{r}}}{r^2} dV'$
	$\mathbf{A} = \frac{\mu}{4\pi} \int_V \frac{\mathbf{J}}{r} dV'$	$\mathbf{A} = \frac{\mu}{c} \int_V \frac{\mathbf{J}}{r} dV'$
Poisson equations	$\nabla^2 \varphi = -\frac{\rho}{\epsilon}$	$\nabla^2 \varphi = -4\pi \frac{\rho}{\epsilon}$
	$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$	$\nabla^2 \mathbf{A} = -\frac{4\pi}{c} \mu \mathbf{J}$

Name of formula	SI (MKSA)	Gaussian
Wave equations	$\nabla^2 \mathbf{E} - \epsilon\mu \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$	$\nabla^2 \mathbf{E} - \frac{\epsilon\mu}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$
	$\nabla^2 \mathbf{H} - \epsilon\mu \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0$	$\nabla^2 \mathbf{H} - \frac{\epsilon\mu}{c^2} \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0$
Dynamic potentials	$\mathbf{B} = \nabla \times \mathbf{A}$	$\mathbf{B} = \nabla \times \mathbf{A}$
	$\mathbf{E} = -\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t}$	$\mathbf{E} = -\nabla\varphi - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}$
Lorentz gauge	$\nabla \cdot \mathbf{A} + \epsilon\mu \frac{\partial \varphi}{\partial t} = 0$	$\nabla \cdot \mathbf{A} + \frac{\epsilon\mu}{c} \frac{\partial \varphi}{\partial t} = 0$
D'Alembert equations	$\nabla^2 \varphi - \epsilon\mu \frac{\partial^2 \varphi}{\partial t^2} = -\frac{\rho}{\epsilon}$	$\nabla^2 \varphi - \frac{\epsilon\mu}{c^2} \frac{\partial^2 \varphi}{\partial t^2} = -4\pi \frac{\rho}{\epsilon}$
	$\nabla^2 \mathbf{A} - \epsilon\mu \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J}$	$\nabla^2 \mathbf{A} - \frac{\epsilon\mu}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\frac{4\pi}{c} \mu \mathbf{J}$
Retarding potentials	$\varphi = \frac{1}{4\pi\epsilon} \int_V \frac{\rho(t-r/c)}{r} dV'$	$\varphi = \frac{1}{\epsilon} \int_V \frac{\rho(t-r/c)}{r} dV'$
	$\mathbf{A} = \frac{\mu}{4\pi} \int_V \frac{\mathbf{J}(t-r/c)}{r} dV'$	$\mathbf{A} = \frac{\mu}{c} \int_V \frac{\mathbf{J}(t-r/c)}{r} dV'$
Energy density	$w = \frac{1}{2} (\mathbf{E} \cdot \mathbf{D} + \mathbf{H} \cdot \mathbf{B})$	$w = \frac{1}{8\pi} (\mathbf{E} \cdot \mathbf{D} + \mathbf{H} \cdot \mathbf{B})$
Poynting vector	$\mathbf{P} = \mathbf{E} \times \mathbf{H}$	$\mathbf{P} = \frac{c}{4\pi} \mathbf{E} \times \mathbf{H}$

### A.3 Prefixes and Symbols for Multiples

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Multiple	Prefix	Symbol
$10^{-18}$	atto	a
$10^{-15}$	femto	f
$10^{-12}$	pico	p
$10^{-9}$	nano	n
$10^{-6}$	micro	$\mu$
$10^{-3}$	milli	m
$10^{-2}$	centi	c
$10^{-1}$	deci	d
10	deka	da
$10^2$	hecto	h
$10^3$	kilo	k
$10^6$	mega	M
$10^9$	giga	G
$10^{12}$	tera	T
$10^{15}$	peta	P
$10^{18}$	exa	E

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# Appendix B

## Vector Analysis

### B.1 Vector Differential Operations

#### B.1.1 General Orthogonal Coordinates

$$u_1, u_2, u_3, h_1, h_2, h_3, h_i = \sqrt{\left(\frac{\partial x}{\partial u_i}\right)^2 + \left(\frac{\partial y}{\partial u_i}\right)^2 + \left(\frac{\partial z}{\partial u_i}\right)^2}, \quad i = 1, 2, 3$$

$$\mathbf{A} = \hat{\mathbf{u}}_1 A_1 + \hat{\mathbf{u}}_2 A_2 + \hat{\mathbf{u}}_3 A_3$$

$$\nabla \varphi = \sum_{i=1}^3 \hat{\mathbf{u}}_i \frac{1}{h_i} \frac{\partial \varphi}{\partial u_i} = \hat{\mathbf{u}}_1 \frac{1}{h_1} \frac{\partial \varphi}{\partial u_1} + \hat{\mathbf{u}}_2 \frac{1}{h_2} \frac{\partial \varphi}{\partial u_2} + \hat{\mathbf{u}}_3 \frac{1}{h_3} \frac{\partial \varphi}{\partial u_3} \quad (\text{B.1})$$

$$\begin{aligned} \nabla \cdot \mathbf{A} &= \frac{1}{h_1 h_2 h_3} \sum_{i=1}^3 \frac{\partial}{\partial u_i} (h_j h_k A_i) \\ &= \frac{1}{h_1 h_2 h_3} \left[ \frac{\partial}{\partial u_1} (h_2 h_3 A_1) + \frac{\partial}{\partial u_2} (h_3 h_1 A_2) + \frac{\partial}{\partial u_3} (h_1 h_2 A_3) \right] \end{aligned} \quad (\text{B.2})$$

$$\begin{aligned} \nabla \times \mathbf{A} &= \sum_{i=1}^3 \hat{\mathbf{u}}_i \frac{1}{h_j h_k} \left[ \frac{\partial}{\partial u_j} (h_k A_k) - \frac{\partial}{\partial u_k} (h_j A_j) \right] \\ &= \hat{\mathbf{u}}_1 \frac{1}{h_2 h_3} \left[ \frac{\partial}{\partial u_2} (h_3 A_3) - \frac{\partial}{\partial u_3} (h_2 A_2) \right] \\ &\quad + \hat{\mathbf{u}}_2 \frac{1}{h_3 h_1} \left[ \frac{\partial}{\partial u_3} (h_1 A_1) - \frac{\partial}{\partial u_1} (h_3 A_3) \right] \\ &\quad + \hat{\mathbf{u}}_3 \frac{1}{h_1 h_2} \left[ \frac{\partial}{\partial u_1} (h_2 A_2) - \frac{\partial}{\partial u_2} (h_1 A_1) \right] \end{aligned} \quad (\text{B.3})$$

$$\begin{aligned}\nabla^2\varphi &= \frac{1}{h_1h_2h_3} \sum_{i=1}^3 \frac{\partial}{\partial u_i} \left( \frac{h_j h_k}{h_i} \frac{\partial \varphi}{\partial u_i} \right) \\ &= \frac{1}{h_1h_2h_3} \left[ \frac{\partial}{\partial u_1} \left( \frac{h_2h_3}{h_1} \frac{\partial \varphi}{\partial u_1} \right) + \frac{\partial}{\partial u_2} \left( \frac{h_3h_1}{h_2} \frac{\partial \varphi}{\partial u_2} \right) + \frac{\partial}{\partial u_3} \left( \frac{h_1h_2}{h_3} \frac{\partial \varphi}{\partial u_3} \right) \right] \quad (\text{B.4})\end{aligned}$$

$$\begin{aligned}\nabla^2 \mathbf{A} &= \nabla(\nabla \cdot \mathbf{A}) - \nabla \times \nabla \times \mathbf{A} \\ &= \hat{\mathbf{u}}_1 \left[ \frac{1}{h_1} \frac{\partial F_0}{\partial u_1} - \frac{1}{h_2h_3} \left( \frac{\partial F_3}{\partial u_2} - \frac{\partial F_2}{\partial u_3} \right) \right] \\ &\quad + \hat{\mathbf{u}}_2 \left[ \frac{1}{h_2} \frac{\partial F_0}{\partial u_2} - \frac{1}{h_3h_1} \left( \frac{\partial F_1}{\partial u_3} - \frac{\partial F_3}{\partial u_1} \right) \right] \\ &\quad + \hat{\mathbf{u}}_3 \left[ \frac{1}{h_3} \frac{\partial F_0}{\partial u_3} - \frac{1}{h_1h_2} \left( \frac{\partial F_2}{\partial u_1} - \frac{\partial F_1}{\partial u_3} \right) \right] \quad (\text{B.5})\end{aligned}$$

where

$$\begin{aligned}F_0 &= \nabla \cdot \mathbf{A} \\ F_1 &= h_1(\nabla \times \mathbf{A})_1 = \frac{h_1}{h_2h_3} \left[ \frac{\partial}{\partial u_2} (h_3A_3) - \frac{\partial}{\partial u_3} (h_2A_2) \right] \\ F_2 &= h_2(\nabla \times \mathbf{A})_2 = \frac{h_2}{h_3h_1} \left[ \frac{\partial}{\partial u_3} (h_1A_1) - \frac{\partial}{\partial u_1} (h_3A_3) \right] \\ F_3 &= h_3(\nabla \times \mathbf{A})_3 = \frac{h_3}{h_1h_2} \left[ \frac{\partial}{\partial u_1} (h_2A_2) - \frac{\partial}{\partial u_2} (h_1A_1) \right]\end{aligned}$$

### B.1.2 General Cylindrical Coordinates

$$\begin{aligned}u_1, u_2, z, \quad h_3 = 1, \quad \frac{\partial h_1}{\partial z} = 0, \quad \frac{\partial h_2}{\partial z} = 0 \\ \nabla\varphi = \hat{\mathbf{u}}_1 \frac{1}{h_1} \frac{\partial \varphi}{\partial u_1} + \hat{\mathbf{u}}_2 \frac{1}{h_2} \frac{\partial \varphi}{\partial u_2} + \hat{\mathbf{z}} \frac{\partial \varphi}{\partial z} \quad (\text{B.6})\end{aligned}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{h_1h_2} \left[ \frac{\partial}{\partial u_1} (h_2A_1) + \frac{\partial}{\partial u_2} (h_1A_2) \right] + \frac{\partial A_z}{\partial z} \quad (\text{B.7})$$

$$\begin{aligned}\nabla \times \mathbf{A} &= \hat{\mathbf{u}}_1 \frac{1}{h_2} \left[ \frac{\partial A_z}{\partial u_2} - \frac{\partial}{\partial z} (h_2A_2) \right] \\ &\quad + \hat{\mathbf{u}}_2 \frac{1}{h_1} \left[ \frac{\partial}{\partial z} (h_1A_1) - \frac{\partial A_z}{\partial u_1} \right] \\ &\quad + \hat{\mathbf{u}}_3 \frac{1}{h_1h_2} \left[ \frac{\partial}{\partial u_1} (h_2A_2) - \frac{\partial}{\partial u_2} (h_1A_1) \right] \quad (\text{B.8})\end{aligned}$$

$$\nabla^2\varphi = \frac{1}{h_1h_2} \left[ \frac{\partial}{\partial u_1} \left( \frac{h_2}{h_1} \frac{\partial \varphi}{\partial u_1} \right) + \frac{\partial}{\partial u_2} \left( \frac{h_1}{h_2} \frac{\partial \varphi}{\partial u_2} \right) \right] + \frac{\partial^2\varphi}{\partial z^2} \quad (\text{B.9})$$

$$\nabla^2 \mathbf{A} = \nabla^2 \mathbf{A}_T + \hat{\mathbf{z}} \nabla^2 A_z \quad (\text{B.10})$$

where  $A_z$  is the longitudinal component and  $\mathbf{A}_T$  is the transverse 2-dimensional vector of  $\mathbf{A}$

$$\begin{aligned} \mathbf{A} &= \mathbf{A}_T + \hat{\mathbf{z}} A_z, & \mathbf{A}_T &= \hat{\mathbf{u}}_1 A_1 + \hat{\mathbf{u}}_2 A_2 \\ \nabla^2 A_z &= \frac{1}{h_1 h_2} \left[ \frac{\partial}{\partial u_1} \left( \frac{h_2}{h_1} \frac{\partial A_z}{\partial u_1} \right) + \frac{\partial}{\partial u_2} \left( \frac{h_1}{h_2} \frac{\partial A_z}{\partial u_2} \right) \right] + \frac{\partial^2 A_z}{\partial z^2} \\ \nabla^2 \mathbf{A}_T &= \hat{\mathbf{u}}_1 \left( \frac{1}{h_1} \frac{\partial F_0}{\partial u_1} - \frac{1}{h_2} \frac{\partial F_z}{\partial u_2} + \frac{\partial^2 A_1}{\partial z^2} \right) \\ &\quad + \hat{\mathbf{u}}_2 \left( \frac{1}{h_2} \frac{\partial F_0}{\partial u_2} + \frac{1}{h_1} \frac{\partial F_z}{\partial u_1} + \frac{\partial^2 A_2}{\partial z^2} \right) \end{aligned} \quad (\text{B.11})$$

where

$$\begin{aligned} F_0 &= \nabla \cdot \mathbf{A}_T = \frac{1}{h_1 h_2} \left[ \frac{\partial}{\partial u_1} (h_2 A_1) + \frac{\partial}{\partial u_2} (h_1 A_2) \right] \\ F_z &= |\nabla \times \mathbf{A}_T| = \frac{1}{h_1 h_2} \left[ \frac{\partial}{\partial u_1} (h_2 A_2) - \frac{\partial}{\partial u_2} (h_1 A_1) \right] \end{aligned}$$

### B.1.3 Rectangular Coordinates

$$\begin{aligned} x, y, z, & \quad h_1 = 1, h_2 = 1, h_3 = 1 \\ \nabla \varphi &= \hat{\mathbf{x}} \frac{\partial \varphi}{\partial x} + \hat{\mathbf{y}} \frac{\partial \varphi}{\partial y} + \hat{\mathbf{z}} \frac{\partial \varphi}{\partial z} \end{aligned} \quad (\text{B.12})$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \quad (\text{B.13})$$

$$\nabla \times \mathbf{A} = \hat{\mathbf{x}} \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{\mathbf{y}} \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{\mathbf{z}} \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \quad (\text{B.14})$$

$$\nabla^2 \varphi = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} \quad (\text{B.15})$$

$$\nabla^2 \mathbf{A} = \hat{\mathbf{x}} \nabla^2 A_x + \hat{\mathbf{y}} \nabla^2 A_y + \hat{\mathbf{z}} \nabla^2 A_z \quad (\text{B.16})$$

### B.1.4 Circular Cylindrical Coordinates

$$\begin{aligned} \rho, \phi, z, & \quad h_1 = 1, h_2 = r, h_3 = 1 \\ \nabla \varphi &= \hat{\boldsymbol{\rho}} \frac{\partial \varphi}{\partial \rho} + \hat{\boldsymbol{\phi}} \frac{1}{\rho} \frac{\partial \varphi}{\partial \phi} + \hat{\mathbf{z}} \frac{\partial \varphi}{\partial z} \end{aligned} \quad (\text{B.17})$$

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \quad (\text{B.18})$$

$$\nabla \times \mathbf{A} = \hat{\rho} \left[ \frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right] + \hat{\phi} \left[ \frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right] + \hat{z} \frac{1}{\rho} \left[ \frac{\partial}{\partial \rho} (\rho A_\phi) - \frac{\partial A_\rho}{\partial \phi} \right] \quad (\text{B.19})$$

$$\nabla^2 \varphi = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \varphi}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 \varphi}{\partial \phi^2} + \frac{\partial^2 \varphi}{\partial z^2} \quad (\text{B.20})$$

$$\nabla^2 \mathbf{A} = \hat{\rho} \left( \nabla^2 A_\rho - \frac{2}{\rho^2} \frac{\partial A_\phi}{\partial \phi} - \frac{A_\rho}{\rho^2} \right) + \hat{\phi} \left( \nabla^2 A_\phi + \frac{2}{\rho^2} \frac{\partial A_\rho}{\partial \phi} - \frac{A_\phi}{\rho^2} \right) + \hat{z} \nabla^2 A_z \quad (\text{B.21})$$

### B.1.5 Spherical Coordinates

$$r, \theta, \phi, \quad h_1 = 1, \quad h_2 = r, \quad h_3 = r \sin \theta$$

$$\nabla \varphi = \hat{r} \frac{\partial \varphi}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial \varphi}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial \varphi}{\partial \phi} \quad (\text{B.22})$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi} \quad (\text{B.23})$$

$$\begin{aligned} \nabla \times \mathbf{A} = & \hat{r} \frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (\sin \theta A_\phi) - \frac{\partial A_\theta}{\partial \phi} \right] \\ & + \hat{\theta} \frac{1}{r} \left[ \frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r} (r A_\phi) \right] + \hat{\phi} \frac{1}{r} \left[ \frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right] \end{aligned} \quad (\text{B.24})$$

$$\nabla^2 \varphi = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \varphi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \varphi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \varphi}{\partial \phi^2} \quad (\text{B.25})$$

$$\begin{aligned} \nabla^2 \mathbf{A} = & \hat{r} \left[ \nabla^2 A_r - \frac{2}{r^2} \left( A_r + \cot \theta A_\theta + \csc \theta \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_\theta}{\partial \theta} \right) \right] \\ & + \hat{\theta} \left[ \nabla^2 A_\theta - \frac{1}{r^2} \left( \csc^2 \theta A_\theta - 2 \frac{\partial A_r}{\partial \theta} + 2 \cot \theta \csc \theta \frac{\partial A_\phi}{\partial \phi} \right) \right] \\ & + \hat{\phi} \left[ \nabla^2 A_\phi - \frac{1}{r^2} \left( \csc^2 \theta A_\phi - 2 \csc \theta \frac{\partial A_r}{\partial \phi} - 2 \cot \theta \csc \theta \frac{\partial A_\theta}{\partial \phi} \right) \right] \end{aligned} \quad (\text{B.26})$$

## B.2 Vector Formulas

### B.2.1 Vector Algebraic Formulas

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A} \quad (\text{B.27})$$

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A} \quad (\text{B.28})$$

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) \quad (\text{B.29})$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C} \quad (\text{B.30})$$

$$(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C}) \quad (\text{B.31})$$

$$(\mathbf{A} \times \mathbf{B}) \times (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \times \mathbf{B} \cdot \mathbf{D})\mathbf{C} - (\mathbf{A} \times \mathbf{B} \cdot \mathbf{C})\mathbf{D} \quad (\text{B.32})$$



## B.2.2 Vector Differential Formulas

$$\nabla(\varphi + \psi) = \nabla\varphi + \nabla\psi \quad (\text{B.33})$$

$$\nabla(\varphi\psi) = \varphi\nabla\psi + \psi\nabla\varphi \quad (\text{B.34})$$

$$\nabla(\mathbf{A} \cdot \mathbf{B}) = (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A} + \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) \quad (\text{B.35})$$

$$\nabla \cdot (\mathbf{A} + \mathbf{B}) = \nabla \cdot \mathbf{A} + \nabla \cdot \mathbf{B} \quad (\text{B.36})$$

$$\nabla \cdot (\varphi\mathbf{A}) = \mathbf{A} \cdot \nabla\varphi + \varphi\nabla \cdot \mathbf{A} \quad (\text{B.37})$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B}) \quad (\text{B.38})$$

$$\nabla \times (\mathbf{A} + \mathbf{B}) = \nabla \times \mathbf{A} + \nabla \times \mathbf{B} \quad (\text{B.39})$$

$$\nabla \times (\varphi\mathbf{A}) = \nabla\varphi \times \mathbf{A} + \varphi\nabla \times \mathbf{A} \quad (\text{B.40})$$

$$\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} \quad (\text{B.41})$$

$$\nabla \cdot \nabla\varphi = \nabla^2\varphi \quad (\text{B.42})$$

$$\nabla \times \nabla\varphi = 0 \quad (\text{B.43})$$

$$\nabla \cdot \nabla \times \mathbf{A} = 0 \quad (\text{B.44})$$

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A} \quad (\text{B.45})$$

## B.2.3 Vector Integral Formulas

Volume  $V$  is bounded by closed surface  $S$ . The unit vector  $\mathbf{n}$  is normal to  $S$  and directed positively outwards.

$$\int_V \nabla\varphi dV = \oint_S \varphi \mathbf{n} dS \quad (\text{B.46})$$

$$\int_V \nabla \cdot \mathbf{A} dV = \oint_S \mathbf{A} \cdot \mathbf{n} dS \quad (\text{Gauss's theorem}) \quad (\text{B.47})$$

$$\int_V \nabla \times \mathbf{A} dV = \oint_S \mathbf{n} \times \mathbf{A} dS \quad (\text{B.48})$$

$$\int_V (\varphi\nabla^2\psi - \nabla\varphi\nabla\psi) dV = \oint_S \varphi\nabla\psi \cdot \mathbf{n} dS \quad (\text{Green's first identity}) \quad (\text{B.49})$$

$$\int_V (\psi\nabla^2\varphi - \varphi\nabla^2\psi) dV = \oint_S (\psi\nabla\varphi - \varphi\nabla\psi) \cdot \mathbf{n} dS \quad (\text{Green's second identity}) \quad (\text{B.50})$$

Open surface  $S$  is bounded by closed line or contour  $l$ .

$$\int_S \mathbf{n} \times \nabla\varphi dS = \oint_l \varphi d\mathbf{l} \quad (\text{B.51})$$

$$\int_S \nabla \times \mathbf{A} \cdot \mathbf{n} dS = \oint_l \mathbf{A} \cdot d\mathbf{l} \quad (\text{Stokes's theorem}) \quad (\text{B.52})$$

### B.2.4 Differential Formulas for the Position Vector

$$\begin{aligned}
 \mathbf{x} &= \hat{x}x + \hat{y}y + \hat{z}z & \mathbf{x}' &= \hat{x}x' + \hat{y}y' + \hat{z}z' \\
 \mathbf{r} = \mathbf{x} - \mathbf{x}' &= \hat{r}r & r = |\mathbf{r}| = |\mathbf{x} - \mathbf{x}'| &= \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} \\
 \nabla &= \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} & \nabla' &= \hat{x}\frac{\partial}{\partial x'} + \hat{y}\frac{\partial}{\partial y'} + \hat{z}\frac{\partial}{\partial z'} \\
 \nabla r &= -\nabla' r = \frac{\mathbf{r}}{r} = \hat{r} & & \text{(B.53)}
 \end{aligned}$$

$$\nabla \frac{1}{r} = -\nabla' \frac{1}{r} = -\frac{\mathbf{r}}{r^3} = -\frac{\hat{r}}{r^2} \quad \text{(B.54)}$$

$$\nabla \cdot \frac{\hat{r}}{r^2} = -\nabla' \cdot \frac{\hat{r}}{r^2} = 4\pi\delta(\mathbf{r}) = 4\pi\delta(\mathbf{x} - \mathbf{x}') \quad \text{(B.55)}$$

$$\nabla^2 \frac{1}{r} = \nabla'^2 \frac{1}{r} = -4\pi\delta(\mathbf{r}) = -4\pi\delta(\mathbf{x} - \mathbf{x}') \quad \text{(B.56)}$$

# Appendix C

## Bessel Functions

### C.1 Power Series Representations

Bessel functions of the first kind:

$$J_n(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!(m+n)!} \left(\frac{x}{2}\right)^{2m+n} \quad (\text{C.1})$$

Bessel functions of the second kind or Neumann functions:

$$\begin{aligned} N_n(x) &= \frac{2}{\pi} \ln \frac{\gamma x}{2} J_n(x) - \frac{1}{\pi} \sum_{m=0}^{n-1} \frac{(n-m-1)!}{m!} \left(\frac{x}{2}\right)^{2m-n} \\ &- \frac{1}{\pi} \sum_{m=0}^{\infty} \frac{(-1)^m}{m!(m+n)!} \left(\frac{x}{2}\right)^{2m+n} \left(1 + \frac{1}{2} + \cdots + \frac{1}{m} + 1 + \frac{1}{2} + \cdots + \frac{1}{m+n}\right) \end{aligned} \quad (\text{C.2})$$

Modified Bessel functions of the first kind:

$$I_n(x) = \sum_{m=0}^{\infty} \frac{1}{m!(m+n)!} \left(\frac{x}{2}\right)^{2m+n} \quad (\text{C.3})$$

Modified Bessel functions of the second kind:

$$\begin{aligned} K_n(x) &= (-1)^{n+1} \ln \frac{\gamma x}{2} I_n(x) - \frac{1}{2} \sum_{m=0}^{n-1} (-1)^m \frac{(n-m-1)!}{m!} \left(\frac{x}{2}\right)^{2m-n} \\ &- \frac{(-1)^n}{2} \sum_{m=0}^{\infty} \frac{1}{m!(m+n)!} \left(\frac{x}{2}\right)^{2m+n} \left(1 + \frac{1}{2} + \cdots + \frac{1}{m} + 1 + \frac{1}{2} + \cdots + \frac{1}{m+n}\right) \end{aligned} \quad (\text{C.4})$$

where  $\gamma$  is the Euler's constant

$$\ln \gamma = \lim_{n \rightarrow \infty} \left( \sum_{m=1}^n \frac{1}{m} - \ln n \right), \quad \ln \gamma = 0.5772, \quad \gamma = 1.781$$

## C.2 Integral Representations

$$J_n(x) = \frac{j^{-n}}{2\pi} \int_0^{2\pi} e^{j(x \cos \alpha - n\alpha)} d\alpha, \quad J_0(x) = \frac{1}{2\pi} \int_0^{2\pi} e^{jx \cos \alpha} d\alpha \quad (\text{C.5})$$

## C.3 Approximate Expressions

### C.3.1 Leading Terms of Power Series (Small Argument)

$$J_0(x) \xrightarrow{x \rightarrow 0} 1 - \frac{x^2}{4}, \quad J_1(x) \xrightarrow{x \rightarrow 0} \frac{x}{2} - \frac{x^3}{16}, \quad J_n(x) \xrightarrow{x \rightarrow 0} \frac{1}{n!} \left(\frac{x}{2}\right)^n \quad (\text{C.6})$$

$$N_0(x) \xrightarrow{x \rightarrow 0} \frac{2}{\pi} \ln \frac{\gamma x}{2}, \quad N_1(x) \xrightarrow{x \rightarrow 0} \frac{2}{\pi x}, \quad N_n(x) \xrightarrow{x \rightarrow 0} \frac{(n-1)!}{\pi} \left(\frac{2}{x}\right)^n, \quad (n \neq 0) \quad (\text{C.7})$$

$$I_0(x) \xrightarrow{x \rightarrow 0} 1 + \frac{x^2}{4}, \quad I_1(x) \xrightarrow{x \rightarrow 0} \frac{x}{2} + \frac{x^3}{16}, \quad I_n(x) \xrightarrow{x \rightarrow 0} \frac{1}{n!} \left(\frac{x}{2}\right)^n \quad (\text{C.8})$$

$$K_0(x) \xrightarrow{x \rightarrow 0} \ln \frac{2}{\gamma x}, \quad K_1(x) \xrightarrow{x \rightarrow 0} \frac{1}{x}, \quad K_n(x) \xrightarrow{x \rightarrow 0} \frac{(n-1)!}{2} \left(\frac{2}{x}\right)^n, \quad (n \neq 0) \quad (\text{C.9})$$

### C.3.2 Leading Terms of Asymptotic Series (Large Argument)

$$J_n(x) \xrightarrow{x \rightarrow \infty} \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{\pi}{4} - \frac{n\pi}{2}\right), \quad N_n(x) \xrightarrow{x \rightarrow \infty} \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{\pi}{4} - \frac{n\pi}{2}\right) \quad (\text{C.10})$$

$$H_n^{(1)}(x) \xrightarrow{x \rightarrow \infty} \sqrt{\frac{2}{\pi x}} e^{j\left(x - \frac{\pi}{4} - \frac{n\pi}{2}\right)}, \quad H_n^{(2)}(x) \xrightarrow{x \rightarrow \infty} \sqrt{\frac{2}{\pi x}} e^{-j\left(x - \frac{\pi}{4} - \frac{n\pi}{2}\right)} \quad (\text{C.11})$$

$$I_n(x) \xrightarrow{x \rightarrow \infty} \frac{1}{\sqrt{2\pi x}} e^x, \quad K_n(x) \xrightarrow{x \rightarrow \infty} \sqrt{\frac{\pi}{2x}} e^{-x} \quad (\text{C.12})$$

## C.4 Formulas for Bessel Functions

$Z_n(x)$  represents  $J_n(x)$ ,  $N_n(x)$ ,  $H_n^{(1)}(x)$ , and  $H_n^{(2)}(x)$ .

### C.4.1 Recurrence Formulas

$$Z_{n-1}(x) + Z_{n+1}(x) = \frac{2n}{x} Z_n(x) \quad (\text{C.13})$$

$$I_{n-1}(x) - I_{n+1}(x) = \frac{2n}{x} I_n(x) \quad (\text{C.14})$$

$$K_{n-1}(x) - K_{n+1}(x) = -\frac{2n}{x} K_n(x) \quad (\text{C.15})$$

### C.4.2 Derivatives

$$Z'_n(x) = \frac{1}{2}[Z_{n-1}(x) - Z_{n+1}(x)] = Z_{n-1}(x) - \frac{n}{x}Z_n(x) = \frac{n}{x}Z_n(x) - Z_{n+1}(x) \tag{C.16}$$

$$I'_n(x) = \frac{1}{2}[I_{n-1}(x) + I_{n+1}(x)] = I_{n-1}(x) - \frac{n}{x}I_n(x) = \frac{n}{x}I_n(x) + I_{n+1}(x) \tag{C.17}$$

$$K'_n(x) = -\frac{1}{2}[K_{n-1}(x) + K_{n+1}(x)] = -K_{n-1}(x) - \frac{n}{x}K_n(x) = \frac{n}{x}K_n(x) - K_{n+1}(x) \tag{C.18}$$

$$Z'_0(x) = -Z_1(x), \quad I'_0(x) = I_1(x), \quad K'_0(x) = -K_1(x) \tag{C.19}$$

$$Z'_1(x) = Z_0(x) - \frac{1}{x}Z_1(x), \quad I'_1(x) = I_0(x) - \frac{1}{x}I_1(x), \quad K'_0(x) = -K_0(x) - \frac{1}{x}K_1(x) \tag{C.20}$$

### C.4.3 Integrals

$$\int x^{n+1}Z_n(x)dx = x^{n+1}Z_{n+1}(x) \tag{C.21}$$

$$\int x^{-(n-1)}Z_n(x)dx = -x^{-(n-1)}Z_{n-1}(x) \tag{C.22}$$

$$\int xZ_n^2(kx)dx = \frac{x^2}{2}[Z_n^2(kx) - Z_{n-1}(kx)Z_{n+1}(kx)] \tag{C.23}$$

### C.4.4 Wronskian

Define  $W[F_1(x), F_2(x)] = F_1(x)\frac{dF_2(x)}{dx} - F_2(x)\frac{dF_1(x)}{dx}$  as the Wronskian.

$$W[J_n(x), N_n(x)] = \frac{2}{\pi x} \tag{C.24}$$

$$W[J_n(x), H_n^{(1)}(x)] = j\frac{2}{\pi x} \tag{C.25}$$

$$W[J_n(x), H_n^{(2)}(x)] = -j\frac{2}{\pi x} \tag{C.26}$$

$$W[H_n^{(1)}(x), H_n^{(2)}(x)] = j\frac{4}{\pi x} \tag{C.27}$$

$$W[I_n(x), K_n(x)] = -\frac{1}{x} \tag{C.28}$$

Consequently

$$J_n(x)N_{n+1}(x) - N_n(x)J_{n+1}(x) = -\frac{2}{\pi x} \tag{C.29}$$

$$I_n(x)K_{n+1}(x) - K_n(x)I_{n+1}(x) = \frac{1}{x} \tag{C.30}$$

## C.5 Spherical Bessel Functions

### C.5.1 Bessel Functions of Order $n + 1/2$

$$J_{n+1/2}(x) = \sqrt{\frac{2}{\pi}} x^{n+1/2} \left(-\frac{1}{x}\right)^n \frac{d^n}{dx^n} \frac{\sin x}{x} \quad (\text{C.31})$$

$$N_{n+1/2}(x) = (-1)^{n+1} \sqrt{\frac{2}{\pi}} x^{n+1/2} \left(\frac{1}{x}\right)^n \frac{d^n}{dx^n} \frac{\cos x}{x} \quad (\text{C.32})$$

$$H_{n+1/2}^{(1)}(x) = \sqrt{\frac{2}{\pi x}} e^{-jn(n+1)/2} e^{jx} \sum_{k=0}^n (-1)^k \frac{(n+k)!}{k!(n-k)!} \frac{1}{(2jx)^k} \quad (\text{C.33})$$

$$H_{n+1/2}^{(2)}(x) = \sqrt{\frac{2}{\pi x}} e^{jn(n+1)/2} e^{-jx} \sum_{k=0}^n \frac{(n+k)!}{k!(n-k)!} \frac{1}{(2jx)^k} \quad (\text{C.34})$$

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x, \quad J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin x}{x} - \cos x\right) \quad (\text{C.35})$$

$$N_{1/2}(x) = -\sqrt{\frac{2}{\pi x}} \cos x, \quad N_{3/2}(x) = -\sqrt{\frac{2}{\pi x}} \left(\sin x + \frac{\cos x}{x}\right) \quad (\text{C.36})$$

$$H_{1/2}^{(1)}(x) = \sqrt{\frac{2}{\pi x}} \frac{e^{jx}}{j}, \quad H_{3/2}^{(1)}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{e^{jx}}{jx} - e^{jx}\right) \quad (\text{C.37})$$

$$H_{1/2}^{(2)}(x) = \sqrt{\frac{2}{\pi x}} \frac{e^{-jx}}{-j}, \quad H_{3/2}^{(2)}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{e^{-jx}}{-jx} - e^{-jx}\right) \quad (\text{C.38})$$

### C.5.2 Spherical Bessel Functions

$$j_n(x) = \sqrt{\frac{\pi}{2x}} J_{n+1/2}(x), \quad n_n(x) = \sqrt{\frac{\pi}{2x}} N_{n+1/2}(x) \quad (\text{C.39})$$

$$h_n^{(1)}(x) = \sqrt{\frac{\pi}{2x}} H_{n+1/2}^{(1)}(x), \quad h_n^{(2)}(x) = \sqrt{\frac{\pi}{2x}} H_{n+1/2}^{(2)}(x) \quad (\text{C.40})$$

### C.5.3 Spherical Bessel Functions by S.A.Schelkunoff

$$\hat{J}_n(x) = \sqrt{\frac{\pi x}{2}} J_{n+1/2}(x), \quad \hat{N}_n(x) = \sqrt{\frac{\pi x}{2}} N_{n+1/2}(x) \quad (\text{C.41})$$

$$\hat{H}_n^{(1)}(x) = \sqrt{\frac{\pi x}{2}} H_{n+1/2}^{(1)}(x), \quad \hat{H}_n^{(2)}(x) = \sqrt{\frac{\pi x}{2}} H_{n+1/2}^{(2)}(x) \quad (\text{C.42})$$

# Appendix D

## Legendre Functions

### D.1 Legendre Polynomials

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n, \quad (\text{D.1})$$

$$Q_n(x) = \frac{1}{2} P_n(x) \ln \frac{1+x}{1-x} - \sum_{l=1}^n \frac{1}{l} P_{l-1}(x) P_{n-l}(x). \quad (\text{D.2})$$

$$P_0(x) = 1, \quad Q_0(x) = \frac{1}{2} \ln \frac{1+x}{1-x} \quad (\text{D.3})$$

$$P_1(x) = x, \quad Q_1(x) = xQ_0(x) - 1 \quad (\text{D.4})$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1), \quad Q_2(x) = P_2(x)Q_0(x) - \frac{3}{2}x \quad (\text{D.5})$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x), \quad Q_3(x) = P_3(x)Q_0(x) - \frac{5}{2}x^2 + \frac{3}{2} \quad (\text{D.6})$$

$$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3), \quad Q_4(x) = P_4(x)Q_0(x) - \frac{35}{8}x^3 + \frac{55}{24}x \quad (\text{D.7})$$

### D.2 Associate Legendre Polynomials

$$P_n^m(x) = (x^2 - 1)^{m/2} \frac{d^m}{dx^m} P_n(x) \quad Q_n^m(x) = (x^2 - 1)^{m/2} \frac{d^m}{dx^m} Q_n(x). \quad (\text{D.8})$$

$$P_1^1(x) = (1 - x^2)^{1/2} \quad (\text{D.9})$$

$$P_2^1(x) = 3(1 - x^2)^{1/2}x \quad (\text{D.10})$$

$$P_3^1(x) = \frac{3}{2}(1 - x^2)^{1/2}(5x^2 - 1) \quad (\text{D.11})$$

$$P_4^1(x) = \frac{5}{2}(1 - x^2)^{1/2}(7x^3 - 3x) \quad (\text{D.12})$$

$$P_2^2(x) = 3(1 - x^2) \quad (\text{D.13})$$

$$P_3^2(x) = 15(1 - x^2)x \quad (\text{D.14})$$

$$P_4^2(x) = \frac{15}{2}(1 - x^2)(7x^2 - 1) \quad (\text{D.15})$$

$$P_3^3(x) = 15(1 - x^2)^{3/2} \quad (\text{D.16})$$

$$P_4^3(x) = 105(1 - x^2)^{3/2}x \quad (\text{D.17})$$

$$P_4^4(x) = 105(1 - x^2)^2 \quad (\text{D.18})$$

## D.3 Formulas for Legendre Polynomials

In the following formulas,  $R_n(x)$  represents  $P_n(x)$  and  $Q_n(x)$ ,  $R_n^m(x)$  represents  $P_n^m(x)$  and  $Q_n^m(x)$  including  $m = 0$ .

### D.3.1 Recurrence Formulas

$$(2n + 1)xR_n^m(x) = (n + m)R_{n-1}^m(x) + (n - m + 1)R_{n+1}^m(x) \quad (\text{D.19})$$

$$2m \frac{x}{\sqrt{1 - x^2}} R_n^m(x) = (n - m - 1)(n + m)R_{n-1}^{m-1}(x) + R_{n+1}^{m+1}(x) \quad (\text{D.20})$$

### D.3.2 Derivatives

$$(2n + 1)R_n(x) = R'_{n+1}(x) - R'_{n-1}(x) \quad (\text{D.21})$$

$$(x^2 - 1)R_n^{m'}(x) = (n - m + 1)R_{n+1}^m(x) - (n + 1)xR_n^m(x) \quad (\text{D.22})$$

### D.3.3 Integrals

$$\int R_n(x)dx = \frac{R_{n+1}(x) - R_{n-1}(x)}{2n + 1} \quad (\text{D.23})$$

$$\int_{-1}^{+1} P_n(x)P_l(x)dx = 0, \quad \text{for } n \neq l \quad (\text{D.24})$$

$$\int_{-1}^{+1} [P_n(x)]^2 dx = \frac{2}{2n + 1}, \quad (\text{D.25})$$

$$\int_{-1}^{+1} P_n^m(x)P_l^m(x)dx = 0, \quad \text{for } n \neq l \quad (\text{D.26})$$

$$\int_{-1}^{+1} [P_n^m(x)]^2 dx = \frac{2}{(2n + 1)} \frac{(n + m)!}{(n - m)!}, \quad (\text{D.27})$$



# Appendix E

## Matrices and Tensors

### E.1 Matrix

1. A *matrix* ( $a$ ) or  $\mathbf{a}$  is a rectangular array of real or complex scalars  $a_{ij}$  which are called the elements of the matrix,

$$\mathbf{a} = (a) = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix} = (a_{ij})_{mn} \quad (\text{E.1})$$

2. A *square matrix* is a matrix with  $m = n$ .
3. A *row matrix* is a matrix with  $m = 1$  and a *Column matrix* is a matrix with  $n = 1$ .
4. The *determinant* of a square matrix is a determinant consisting of the elements of the matrix, and is denoted by  $|A|$  or  $|(A)|$ .
5. The *complementary minor* is the determinant of the sub-matrix obtained from the square matrix ( $A$ ) by deleting the  $i$ th row and the  $j$ th column, and is denoted by  $M_{ij}$ .
6. The *algebraic complement*  $A_{ij}$  is defined as

$$A_{ij} = (-1)^{i+j} M_{ij}$$

7. A *diagonal matrix* is a square matrix whose off-diagonal elements are zero, i.e.,  $a_{ij} = 0$ ,  $i \neq j$ .

8. A *unit matrix* ( $I$ ) is a diagonal matrix where  $a_{ii} = 1$ ; consequently,  $|I| = 1$ .
9. A *zero matrix* ( $0$ ) is the matrix where  $a_{ij} = 0$ , for all  $i, j$ .

## E.2 Matrix Algebra

### E.2.1 Definitions

1. Addition. The sum of two matrices exists only if the two matrices have the same size, i.e., the same number of rows and columns,

$$(A) + (B) = (a_{ij})_{mn} + (b_{ij})_{mn} = (a_{ij} + b_{ij})_{mn} \quad (\text{E.2})$$

2. Multiplication. If  $\alpha$  is a scalar

$$\alpha(A) = (\alpha a_{ij})_{mn} \quad (\text{E.3})$$

3. Product of matrices. The product of two matrices exists only if the number of columns in ( $A$ ) equals the number of rows in ( $B$ ), i.e.,  $(A) = (a_{ij})_{mp}$ ,  $(B) = (b_{ij})_{pn}$ ,

$$(A)(B) = (c_{ij})_{mn} \quad (\text{E.4})$$

where  $i = 1, 2 \cdots m$ ,  $j = 1, 2 \cdots n$ , and

$$c_{ij} = \sum_{k=1}^p a_{ik}b_{kj} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{ip}b_{pj}$$

### E.2.2 Matrix Algebraic Formulas

In the following expressions,  $(A) = (a_{ij})_{mn}$ ,  $(B) = (b_{ij})_{mn}$ ,  $(C) = (c_{ij})_{mn}$  are matrices and  $\alpha$  and  $\beta$  are scalars.

$$(A) + (B) = (B) + (A) \quad (\text{E.5})$$

$$[(A) + (B)] + (C) = (A) + [(B) + (C)] \quad (\text{E.6})$$

$$\alpha[(A) + (B)] = \alpha(A) + \alpha(B) \quad (\text{E.7})$$

$$(\alpha + \beta)(A) = \alpha(A) + \beta(A) \quad (\text{E.8})$$

$$(A)(B) \neq (B)(A) \quad (\text{E.9})$$

$$\alpha[(A)(B)] = [\alpha(A)](B) = (A)[\alpha(B)] \quad (\text{E.10})$$

$$[(A)(B)](C) = (A)[(B)(C)] \quad (\text{E.11})$$

$$[(A) + (B)](C) = (A)(C) + (B)(C) \quad (\text{E.12})$$

$$(A)[(B) + (C)] = (A)(B) + (A)(C) \tag{E.13}$$

$$|(A)(B)| = |A||B| \tag{E.14}$$

$$|\alpha(A)| = \alpha^n |A| \tag{E.15}$$

$$(A) + (0) = (A) \tag{E.16}$$

$$(0)(A) = (A)(0) = (0) \tag{E.17}$$

$$(\mathcal{I})(A) = (A)(\mathcal{I}) = (A) \tag{E.18}$$

### E.3 Matrix Functions

1. The *negative* of a matrix  $(A)$  is the matrix obtained by taking the negative value of all elements of  $(A)$ , and is denoted by  $-(A)$ ,

$$-(A) = (-a_{ij})_{mn}, \quad \text{and} \quad (A) + (-(A)) = 0, \quad -(-(A)) = (A) \tag{E.19}$$

2. The *conjugate* of a matrix  $(A)$  is the matrix obtained by taking the conjugate of all elements of  $(A)$ , and is denoted by  $(A)^*$ ,

$$(A)^* = (a_{ij}^*)_{mn} \tag{E.20}$$

$$((A)^*)^* = (A) \tag{E.21}$$

$$((A)(B))^* = (A)^*(B)^* \tag{E.22}$$

3. The *transpose* of a matrix  $(A)$  is the matrix obtained by interchanging the rows of  $(A)$  and the columns of  $(A)$  and vice versa, and is denoted by  $(A)^T$ ,

$$(A)^T = (a_{ji})_{nm} \tag{E.23}$$

$$((A)^T)^T = (A) \tag{E.24}$$

$$((A)(B))^T = (B)^T(A)^T \tag{E.25}$$

$$(A)^{*T} = (A)^{T*} \tag{E.26}$$

4. The *conjugate transpose* of a matrix  $(A)$  is the matrix obtained by applying the conjugate and transpose operations on  $(A)$  simultaneously, and is denoted by  $(A)^\dagger$ ,

$$(A)^\dagger = (A)^{*T} \tag{E.27}$$

$$((A)(B))^\dagger = (B)^\dagger(A)^\dagger \tag{E.28}$$

5. The *adjoint* of a square matrix  $(A)$  is a square matrix whose elements is equal to the elements of the algebraic complement  $A_{ij}$  in the matrix  $(A)$ , and is denoted by  $(A)^a$ ,

$$(A)^a = (A_{ij})_{nn} \tag{E.29}$$

$$(A)(A)^a = |A|(\mathcal{I}) \tag{E.30}$$

6. The *inverse* of a matrix ( $A$ ) is the matrix that the product of ( $A$ ) and its inverse matrix is a unit matrix. The matrix ( $A$ ) has a unique inverse only if it is square and nonsingular, and is denoted by  $(A)^{-1}$ ,

$$(A)(A)^{-1} = (A)^{-1}(A) = (\mathcal{I}) \quad (\text{E.31})$$

$$(A)^{-1} = \frac{1}{|A|}(A)^a = \frac{1}{|A|}(A_{ij})_{nn} \quad (\text{E.32})$$

$$((A)^{-1})^{-1} = (A) \quad (\text{E.33})$$

$$(\alpha(A))^{-1} = \frac{1}{\alpha}(A)^{-1} \quad (\text{E.34})$$

$$((A)(B))^{-1} = (B)^{-1}(A)^{-1} \quad (\text{E.35})$$

$$((A)^*)^{-1} = ((A)^{-1})^*, \quad ((A)^T)^{-1} = ((A)^{-1})^T, \quad ((A)^\dagger)^{-1} = ((A)^{-1})^\dagger \quad (\text{E.36})$$

## E.4 Special Matrices

1. *Real matrix*,

$$(A)^* = (A), \quad \text{all } a_{ij} \text{ are real}$$

2. *Symmetric matrix*,

$$(A)^T = (A), \quad a_{ij} = a_{ji}$$

3. *Skew-symmetric matrix*,

$$(A)^T = -(A), \quad a_{ij} = 0, \text{ for } i = j, \quad a_{ij} = -a_{ji}, \text{ for } i \neq j$$

4. *Hermitian matrix*,

$$(A)^\dagger = (A), \quad (A)^T = (A)^*$$

5. *Skew-Hermitian matrix*,

$$(A)^\dagger = -(A), \quad (A)^T = -(A)^*$$

6. *Unitary matrix* or *U matrix*,

$$(A)^\dagger(A) = (\mathcal{I}), \quad (A)^\dagger = (A)^{-1}, \quad (A) = ((A)^\dagger)^{-1}$$

7. *Orthogonal matrix* is a real unitary matrix,

$$(A)^T(A) = (\mathcal{I}), \quad (A)^T = (A)^{-1}, \quad (A) = ((A)^T)^{-1}$$

## E.5 Tensors and vectors

1. A *vector* can be expressed by a row matrix or column matrix with three elements, and is denoted by a italic bold-face letter  $\mathbf{A}$ ,

$$\mathbf{A} = [A_x \ A_y \ A_z] = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \quad (\text{E.37})$$

2. A *tensor* of rank 2 can be expressed by a  $3 \times 3$  square matrix and is denoted by a bold face letter  $\mathbf{a}$ ,

$$\mathbf{a} = \begin{bmatrix} a_{xx} & a_{xy} & a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{bmatrix} \quad (\text{E.38})$$

3. Vector and tensor operations

$$\mathbf{a} \cdot \mathbf{A} = \begin{bmatrix} a_{xx} & a_{xy} & a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \quad (\text{E.39})$$

$$\mathbf{A} \cdot \mathbf{a} = [A_x \ A_y \ A_z] \begin{bmatrix} a_{xx} & a_{xy} & a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{bmatrix} \quad (\text{E.40})$$

$$\mathbf{a} \cdot \mathbf{b} = \begin{bmatrix} a_{xx} & a_{xy} & a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{bmatrix} \begin{bmatrix} b_{xx} & b_{xy} & b_{xz} \\ b_{yx} & b_{yy} & b_{yz} \\ b_{zx} & b_{zy} & b_{zz} \end{bmatrix} \quad (\text{E.41})$$

$$\mathbf{A} \cdot \mathbf{a} \cdot \mathbf{A}^* = \mathbf{A}^* \cdot \mathbf{a}^T \cdot \mathbf{A} \quad (\text{E.42})$$

$$\mathbf{A} \cdot \mathbf{a}^* \cdot \mathbf{A}^* = \mathbf{A}^* \cdot \mathbf{a}^\dagger \cdot \mathbf{A} \quad (\text{E.43})$$

$$\mathbf{A} \cdot \mathbf{a} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{a}^T \cdot \mathbf{A} \quad (\text{E.44})$$

$$\mathbf{A} \cdot \mathbf{a}^* \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{a}^\dagger \cdot \mathbf{A} \quad (\text{E.45})$$

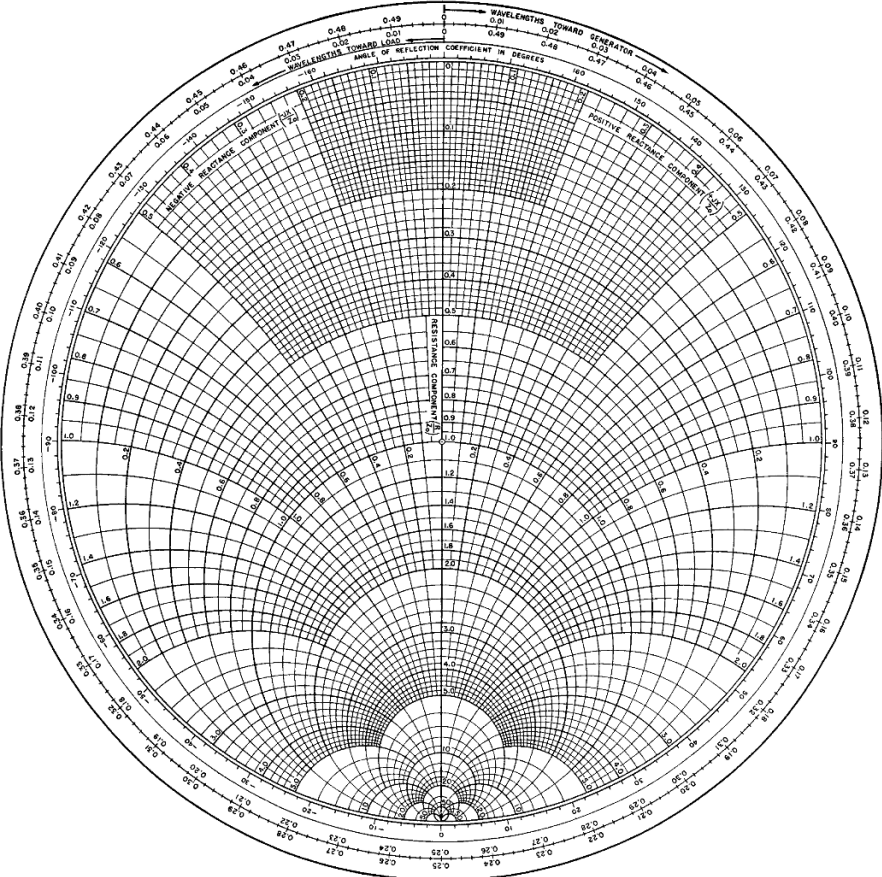
# Physical Constants

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Physical constant	Symbol	
Speed of light in vacuum	$c$	299792458 m/s $\approx 3 \times 10^8$ m/s
Vacuum permittivity	$\epsilon_0$	$\frac{1}{4\pi c^2} \times 10^7$ F/m $\approx 8.85418782 \times 10^{-12}$ F/m
Vacuum permeability	$\mu_0$	$4\pi \times 10^{-7}$ H/m $\approx 12.5663706 \times 10^{-7}$ H/m
Vacuum wave impedance	$\eta_0$	$4\pi c \times 10^{-7}$ $\Omega$ $\approx 120\pi$ $\Omega$
Electron charge magnitude	$e$	$1.6021892 \times 10^{-19}$ C
Electron rest mass	$m$	$9.109534 \times 10^{-31}$ kg
Electron charge to mass ratio	$e/m$	$1.75883 \times 10^{11}$ C/kg
Proton rest mass	$m$	$1.6726485 \times 10^{-27}$ kg
Gyromagnetic ratio	$\gamma$	$1.75883 \times 10^{11}$ rad/(s·T)

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# Smith Chart



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