EXERCISE / HOMEWORK SET 01

ÚFV/TRANS/18 - Transport Properties of Solids lecture by Martin Gmitra Summer Semester 2024/2025

Exercise:

- 1. Derive magnetoresistivity of classical Hall effect assuming that current density $j_i = \sum_j \sigma_{ij}(B)E_j$, where $\sigma_{ij}(B)$ are the components of magnetoconductivity tensor that depends on magnetic field *B*. Find Hall coefficient and Hall potential. Discuss limiting cases $B \to \infty$ for isotropic one-band and two-band model [follow book by G. Grosso, G. P. Parravicini p. 662-668].
- 2. Derive conductance from simplified Boltzmann equation, where at small electric fields the Fermi surface in *k*-space is displaced by $e\tau \mathbf{E}/\hbar$ and electron velocity $\mathbf{v}(\mathbf{k}) = \hbar \mathbf{k}/m^*$, and show for low temperature limit assuming free-electrons that the conductance reduces to Drude's formula [follow book by T. Heinzel p. 44].

Homework:

- 1. [1 point] Find within the Drude classical model the velocity v(t) of electrons for $t \gg \tau$ accelerated in homogeneous electric field **E** and zero magnetic field **B** = 0 within the Lorentz force assuming the solution in the form $v(t) = a + be^{-t/\tau}$.
- 2. [1 point] Show that $\psi_{\mathbf{k}}(\mathbf{r},t)$ obeys Bloch's theorem if $\psi_{\mathbf{k}}(\mathbf{r},t)$ evolves in time as $\psi_{\mathbf{k}}(\mathbf{r},t) = \exp(-iHt/\hbar)\psi_{\mathbf{k}}(\mathbf{r})$, where Hamiltonian $H = H_0 + e\mathbf{E} \cdot \mathbf{r}$ and H_0 is Hamiltonian at t = 0 and \mathbf{E} is the electric field.
- 3. *[1 point]* Give an argument why completely filled band can not contribute to an electric current.
- 4. Consider density of atoms of $8.5 \times 10^{28} m^{-3}$, in a crystal lattice (that is made of cooper atoms), where each atom donates 1 electron in the conduction band.
 - a) [1 point] Assuming that the effective mass of the conduction electrons is the same as the free electron mass, calculate the Fermi energy [express it in eV].
 - b) [1 point] Assume now that the electrons participate in the current flow if their energies correspond to the occupancy $n(\epsilon)$ that is not too close to 1 (no empty states available for the accelerated electrons) and not too small (no electrons to accelerate). At T = 300 K, calculate the energy interval that is occupied by the electrons that participate in the current flow, assuming that for these electrons the occupancy varies between 0.1 and 0.9.
- 5. [1 point] Have a look at the Fermi surface of aluminum at http://www.phys.ufl.edu/fermisurface/. Does the surface look spherical? Can we consider it as approximately spherical (see the construction in Fig. 15.14 on page 301 of Ashcroft-Mermin book)? How many bands contribute to this Fermi surface?
- 6. *[4 points]* Derive electrical conductance (similar as in exercise 2) but for the energy dispersion given in two-dimensions by $\epsilon(\mathbf{k}) = \hbar^2 (k_x^2 + k_y^2)/(2m^*) + \alpha k_y$.