EXERCISE / HOMEWORK SET 01
ÚFV/TRANS/18 - Transport Properties of Solids
lecture by Martin Gmitra
Summer Semester 2020/2021

Exercise:

1. Derive conductance from simplified Boltzmann equation, where at small electric fields the Fermi surface in *k*-space is displaced by $e\tau\mathbf{E}/\hbar$ and electron velocity $\mathbf{v}(\mathbf{k}) = \hbar\mathbf{k}/m^*$, and show for low temperature limit assuming free-electrons that the conductance reduces to Drude's formula [*follow book by T. Heinzel p. 44*].

2. Derive electrical conductance homogeneous metal assuming homogeneous electric field and relaxation time (independent on energy and temperature). How would look like the conductance for energy dispersion $\epsilon(\mathbf{k}) = \hbar^2(k_x^2 + k_y^2)/(2m^*)$ [*follow book by V. Ilkovič p. 241 task VI.1*].

Homework:

1. *[1 point]* Find within the Drude classical model the velocity $v(t)$ of electrons for $t \gg \tau$ accelerated in homogeneous electric field $\mathbf{E}$ and zero magnetic field $\mathbf{B} = 0$ within the Lorentz force assuming the solution in the form $v(t) = a + be^{-t/\tau}$.

2. *[1 point]* Show that $\psi_\mathbf{k}(\mathbf{r}, t)$ obeys Bloch's theorem if $\psi_\mathbf{k}(\mathbf{r}, t)$ evolves in time as $\psi_\mathbf{k}(\mathbf{r}, t) = \exp(-iHt/\hbar)\psi_\mathbf{k}(\mathbf{r})$, where Hamiltonian $H = H_0 + e\mathbf{E}\cdot\mathbf{r}$ and H_0 is Hamiltonian at $t = 0$ and $\mathbf{E}$ is the electric field.

3. Consider density of atoms of $8.5 \times 10^{28}\mathrm{m}^{-3}$, in a crystal lattice (that is made of cooper atoms), where each atom donates 1 electron in the conduction band.
   a) *[1 point]* Assuming that the effective mass of the conduction electrons is the same as the free electron mass, calculate the Fermi energy [express it in eV].
   b) *[1 point]* Assume now that the electrons participate in the current flow if their energies correspond to the occupancy $n(\epsilon)$ that is not too close to 1 (no empty states available for the accelerated electrons) and not too small (no electrons to accelerate). At $T = 300\,\mathrm{K}$, calculate the energy interval that is occupied by the electrons that participate in the current flow, assuming that for these electrons the occupancy varies between 0.1 and 0.9.

4. *[1 point]* Have a look at the Fermi surface of aluminum at http://www.phys.ufl.edu/fermisurface/. Does the surface look spherical? Can we consider it as approximately spherical (*see the construction in Fig. 15.14 on page 301 of Ashcroft-Mermin book)*? How many bands contribute to this Fermi surface?

5. *[1 point]* Read from Chapter 12 (pages 214-223) in Ashcroft-Mermin book about semiclassical model and wavepacket dynamics. Write a short essay (up to one page) about main facts what you learned.

6. *[4 points]* Derive electrical conductance (similar as in exercise 2) but for the energy dispersion given in two-dimensions by $\epsilon(\mathbf{k}) = \hbar^2(k_x^2 + k_y^2)/(2m^*) + \alpha k_y$.