EXERCISE / HOMEWORK SET 03
ÚFV/TRANS/18 - Transport Properties of Solids
lecture by Martin Gmitra
Summer Semester 2020/2021

Exercise:

1. Demonstrate calculation of density of states for arbitrary energy dispersion in one-dimensional case and show that is equal to $\rho_{\mathrm{1D}} = \dfrac{2}{\pi}\dfrac{1}{\hbar v_k}$.

2. Show that current operator has two components the diamagnetic term and the paramagnetic term [*follow book by H. Bruus, K. Flensberg Chap. 1.4.3, p. 21, Eq. (1.98)*].

Homework:

1. *[1 point]* Write Fourier transform of the particle-current density $\mathbf{j}_{\mathrm{i}}(\mathbf{r}) = \dfrac{1}{2m_e i}(\Psi^\dagger(\mathbf{r})\nabla\Psi(\mathbf{r}) - \Psi(\mathbf{r})\nabla\Psi^\dagger(\mathbf{r}))$ and write it also for free particles. For the field operator consider the following expansion $\Psi(\mathbf{r}) = \sum_\lambda c_\lambda \phi_\lambda(\mathbf{r})$, $\Psi^\dagger(\mathbf{r}) = \sum_\lambda c_\lambda^\dagger \phi_\lambda^*(\mathbf{r})$.

2. Consider two-dimensional metal with a square lattice of lattice constant spacing a. The conduction band dispersion is given in tight-binding approximation $\epsilon(k) = \epsilon_0 + t(2 - \cos(k_x a) - \cos(k_y a))$.
   a) *[2 points]* Calculate the conductivity tensor using the solution of the Boltzmann equation
   b) *[1 points]* Compare the above result to the conductivity from the Drude model assuming the same electron density and relaxation time. How does renormalize effective mass?
   c) *[2 points]* Determine the Hall coefficient for the nearly empty band, half-filled band, and nearly full band assuming for the relaxation time τ is independent of the electron momentum.

3. *[1 point]* Considering a spin-degenerate 2D electron gas in thermal equilibrium characterized by temperature T and chemical potential μ, find the electron density $n(T,\mu)$ and compressibility $\chi(T,\mu) = \partial n/\partial\mu$. Using their general form as an integral over energy, find explicit expressions for two limits: (i) degenerate Fermi gas, $\mu \gg T$ and (ii) non-degenerate Boltzmann gas, $|\mu| \gg T$, $\mu < 0$.

4. *[1 point]* Show that Sharvin conductance of a two-dimensional classical point contact of width w is equal $G_{\mathrm{S}}^{\mathrm{2D}} = \dfrac{2e^2}{h}\dfrac{wk_{\mathrm{F}}}{\pi}$.

5. *[2 points]* An electron moving through the crystal can be described by a superposition of Bloch waves. Consider the time-dependent, one-dimensional Bloch wave functions $\psi_k(x,t) = e^{ikx - \omega(k)t}u_k(x)$ and the wave packet
   $\Psi(x,t) = \displaystyle\int_{-\infty}^{\infty} e^{-(k-k_0)^2/b^2} e^{ikx-\omega(k)t} u_k(x)\mathrm{d}k.$
   If b is much smaller than the size of the Brillouin zone, the Gauss function $e^{-(k-k_0)^2/b^2}$ is strongly peaked around k_0, and we can assume that $u_k(x) = u_{k_0}(x)$ does not depend on k and that the dispersion of the electronic states $\omega(k) = E(k)\hbar$ is linear, that is, $\omega(k) = v_g(k-k_0) + \omega(k_0) = v_g k + \omega_0$ with $\omega_0 = \omega(k_0)v_g k_0$, and $v_g = \partial\omega/\partial k$ at k_0. Show that the maximum of the probability distribution associated with this wave packet moves with a velocity v_g.