

EXERCISE / HOMEWORK SET 03
 ÚFV/TRANS/18 - Transport Properties of Solids
 lecture by Martin Gmitra
 Summer Semester 2022/2023

Exercise:

1. Demonstrate calculation of density of states for two-dimensional electron gas confined in a strip of width w .
2. Show that current operator has two components the diamagnetic term and the paramagnetic term [follow book by H. Bruus, K. Flensberg Chap. 1.4.3, p. 21, Eq. (1.98)].

Homework:

1. [1 point] Write Fourier transform of the particle-current density $\mathbf{j}_i(\mathbf{r}) = \frac{1}{2m_e i} (\Psi^\dagger(\mathbf{r}) \nabla \Psi(\mathbf{r}) - \Psi(\mathbf{r}) \nabla \Psi^\dagger(\mathbf{r}))$ and write it also for free particles. For the field operator consider the following expansion $\Psi(\mathbf{r}) = \sum_{\lambda} c_{\lambda} \phi_{\lambda}(\mathbf{r})$, $\Psi^\dagger(\mathbf{r}) = \sum_{\lambda} c_{\lambda}^\dagger \phi_{\lambda}^*(\mathbf{r})$.
2. Consider two-dimensional metal with a square lattice of lattice constant spacing a . The conduction band dispersion is given in tight-binding approximation $\epsilon(k) = \epsilon_0 + t(2 - \cos(k_x a) - \cos(k_y a))$.
 - a) [2 points] Calculate the conductivity tensor using the solution of the Boltzmann equation
 - b) [1 points] Compare the above result to the conductivity from the Drude model assuming the same electron density and relaxation time. How does renormalize effective mass?
 - c) [2 points] Determine the Hall coefficient for the nearly empty band, half-filled band, and nearly full band assuming for the relaxation time τ is independent of the electron momentum.
3. [1 point] Considering a spin-degenerate 2D electron gas in thermal equilibrium characterized by temperature T and chemical potential μ , find the electron density $n(T, \mu)$ and compressibility $\chi(T, \mu) = \partial n / \partial \mu$. Using their general form as an integral over energy, find explicit expressions for two limits: (i) degenerate Fermi gas, $\mu \gg T$ and (ii) non-degenerate Boltzmann gas, $|\mu| \gg T$, $\mu < 0$.
4. [1 point] Show that Sharvin conductance of a two-dimensional classical point contact of width w is equal $G_S^{2D} = \frac{2e^2}{h} \frac{wk_F}{\pi}$.
5. [2 points] An electron moving through the crystal can be described by a superposition of Bloch waves. Consider the time-dependent, one-dimensional Bloch wave functions $\psi_k(x, t) = e^{ikx - \omega(k)t} u_k(x)$ and the wave packet $\Psi(x, t) = \int_{-\infty}^{\infty} e^{-(k-k_0)^2/b^2} e^{ikx - \omega(k)t} u_k(x) dk$.
 If b is much smaller than the size of the Brillouin zone, the Gauss function $e^{-(k-k_0)^2/b^2}$ is strongly peaked around k_0 , and we can assume that $u_k(x) = u_{k_0}(x)$ does not depend on k and that the dispersion of the electronic states $\omega(k) = E(k)/\hbar$ is linear, that is, $\omega(k) = v_g(k - k_0) + \omega(k_0) = v_g k + \omega_0$ with $\omega_0 = \omega(k_0)$, and $v_g = \partial \omega / \partial k$ at k_0 . Show that the maximum of the probability distribution associated with this wave packet moves with a velocity v_g .