# EXERCISE / HOMEWORK SET 05

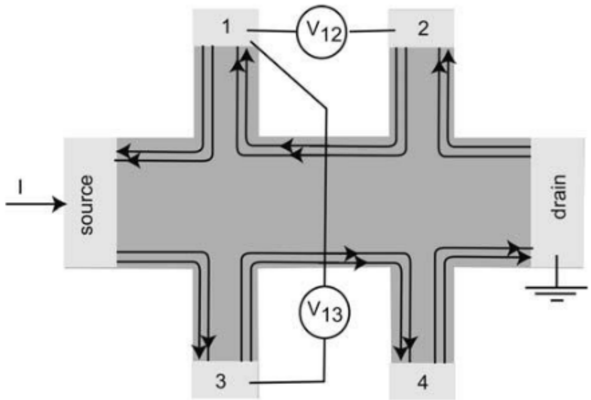
ÚFV/TRANS/18 - Transport Properties of Solids

lecture by Martin Gmitra

Summer Semester 2020/2021

**Exercise:**

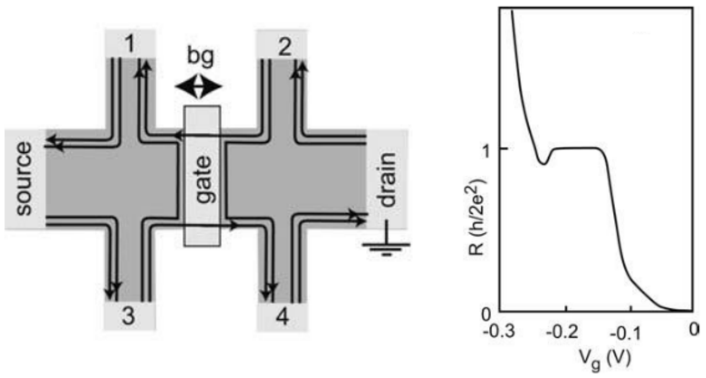
1. By means of Landauer-Büttiker formalism set up matrix equation for the current flowing in one-dimensional edge states in Hall bar exposed in a strong magnetic field and find longitudinal and Hall resistances [*see e.g. book T. Heinzel page 199*].




2. Find scattering matrix in the presence of time-reversal symmetry for three-terminal junction assuming that each lead has a single mode and it is symmetric (the lead indices are interchangeable).

**Homework:**

1. Consider multiterminal Hall bar device with a gate stripe which allows to tune by gate voltage $V_\mathrm{g}$ the electron density and thus the number of occupied Landau levels underneath. If the number of channels under the gate is smaller than outside the gated area the edge states can be redirected at the gate. Assume that there are $N$ channels in the ungated region and $M$ in the gated region, $M \leq N$, transmission probability is given by the average value $T$ per channels, and spin degeneracy is lifted in all the edge states.




a) *[2 points]* Write down matrix equation relating current and voltage using Landauer-Büttiker formalism.
b) *[1 point]* Reduce the system of equation considering zero for the drain potential and use the fact for the source and drain current $I_\mathrm{s} + I_\mathrm{d} = 0$.
c) *[2 points]* Calculate resistances $R_{ij}$ for $i, j = 1, \ldots, 4$. Explain experimentally observed plateau in longitudinal $R_{xx} = h/2e^2$.

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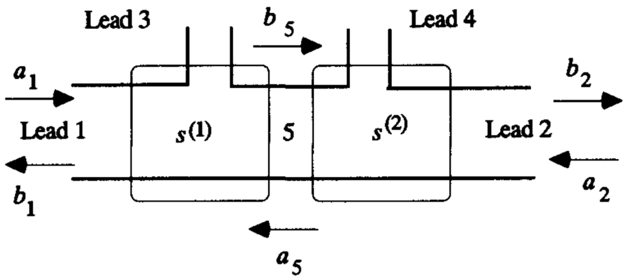
2. *[2 points]* Consider a four-terminal device as shown below and assume division of the system into two parts. The S-matrices $s^{(1)}$ and $s^{(2)}$ of the individual sections relating the outgoing wave amplitudes to the incoming ones have the following forms

$$\begin{pmatrix} b_{13} \\ b_5 \end{pmatrix} = \begin{bmatrix} r^{(1)} & t'^{(1)} \\ t^{(1)} & r'^{(1)} \end{bmatrix} \begin{pmatrix} a_{13} \\ a_5 \end{pmatrix} \text{and} \begin{pmatrix} a_5 \\ b_{24} \end{pmatrix} = \begin{bmatrix} r^{(2)} & t'^{(2)} \\ t^{(2)} & r'^{(2)} \end{bmatrix} \begin{pmatrix} b_5 \\ a_{24} \end{pmatrix}.$$

They can be combined into overall S-matrix $\begin{pmatrix} b_{13} \\ b_{24} \end{pmatrix} = \begin{bmatrix} r & t' \\ t & r' \end{bmatrix} \begin{pmatrix} a_{13} \\ a_{24} \end{pmatrix}$. Find the components of the overall S-matrix in terms of the corresponding individual sections.




3. *[2 points]* In the integer quantum Hall effect (IQHE) the longitudinal resistance vanishes $\rho_{xx} \approx 0$, while the transverse resistance $\rho_{xy} = h/(2e^2N)$ is quantized. Here we assume spin degeneracy (the factor 2) and $N$ propagating modes in the lead at the given Fermi level and magnetic field. What is the conductance tensor, longitudinal and transverse conductances?

4. *[1 point]* Consider the quasiclassical expression for the resistivity tensor in a magnetic field and show that in the magnetic field corresponding to exactly $N$ Landau levels filled, the transverse resistivity is the same as measured on the plauteaus. Does this simple calculation explain IQHE?