

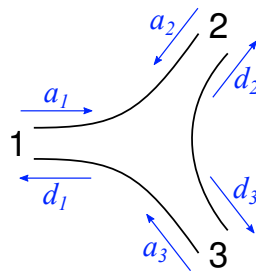
EXERCISE / HOMEWORK SET 06
 ÚFV/TRANS/18 - Transport Properties of Solids
 lecture by Martin Gmitra
 Summer Semester 2020/2021

Exercise:

1. Learn about Laughlin's thought experiment demonstrating the robustness of IQHE, in C. Kittel book "Introduction to Solid State Physics", Wiley, 1996, 7th ed., page 568.
2. Find the S -matrix elements in terms of the transfer matrix M for a general 1d scattering problem.
3. Discuss Aharonov-Bohm effect in terms of the S -matrix studying transmission of a quantum ring in magnetic field. Follow book by T. Heinzel "Mesoscopic Electronics in Solid State Nanostructures", page 241.

Homework:

1. [2 points] Consider the following measuring setup of a quantum point contact in the IQHE regime. (HW06 t3)
2. Consider the scattering geometry of a mesoscopic beam splitter where regions 2 and 3 are assumed to be identical.



- a) [2 points] Show that the S -matrix can be described as follows $S = \begin{pmatrix} r_0 & t & t \\ t & r & r' \\ t & r' & r \end{pmatrix}$
- b) [2 points] Using the unitarity of the S -matrix and assuming that all the parameters are real show the following $t^2 = (1 - r_0^2)/2$, $r = \mp(1 + r_0)/2$, $r' = \pm(1 + r_0)/2$. What are the limiting values for t^2 ?
3. [3 points] Suppose an island (quantum dot) being charged in addition with a background charge q_b which is generally not quantized. Discuss stability conditions for the uncharged island ($n = 0$). How presence of q_b affects the Coulomb blockade? Draw the energy diagrams for zero voltage $V = 0$. Discuss case when $q_b = e/2$.
4. [1 point] The single electron tunneling rate is given by $\Gamma_{1+}(n) = \frac{h}{e} G_T \frac{\Delta E_{1+}(n)}{\exp[\Delta E_{1+}(n)/k_B T] - 1}$, where G_T is the tunnel conductance, and $\Delta E_{1+}(n)$ is the charging energy of the island. Show that the tunneling rates between the electrode and the island in the Coulomb-blockade system satisfy the detailed balance criterion $\frac{\Gamma_{1+}(n)}{\Gamma_{1-}(n+1)} = \exp[-\Delta E_{1+}(n)/k_B T]$.